SPECTRUM SENSING USING BUSSGANG THEOREM FOR IEEE 802.22 WRAN

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ABSTRACT

Utilization problem of the limited spectrum is one of the most important issues in wireless communication systems. Cognitive radio technique which is finding and utilizing frequency holes is also one of those techniques. Specially, the spectrum sensing technique to detect the primary user signal is a core technology in cognitive radio area. In this paper, we propose the spectrum sensing algorithm using Bussgang theorem. The proposed algorithm calculates the statistical difference between the Gaussian noise and the primary user signal by applying Bussgang theorem to the received signal. The algorithm is not affected by noise uncertainty and can detect the primary user signal in the very low SNR (signal-to-noise power ratio) environment. We evaluate the algorithm through computer simulations with 12 ATSC A/74 DTV signal captures based on IEEE 802.22 WRAN (wireless regional area network)

I. INTRODUCTION

Most of the usable spectrum is already allocated to conventional systems, but the allocated spectrum is not efficiently used. Much of the allocated spectrum is unused in the certain time and area. The FCC (federal communication commission) reported that there are temporal and geographic variations in the usage of allocated spectrum ranging from 15% to 85% [1]. In 2004, the IEEE formed 802.22 WG (working group) to establish an international standard for WRAN (wireless regional area networks) based on CR (cognitve radio) [2].

Basic concept of the WRAN system is that WRAN system detects primary user, identifies idle channels, exploits these channels for a broadband wireless communication, and avoid primary users. For this basic concept, spectrum sensing is one of the most important components of the WRAN system. Sensing Tiger Team of the WRAN WG for the evaluation of proposed sensing algorithms require an accurate sensing method satisfying detection of primary user signal when the SNR (signal to noise ratio) is below -20dB.

Some sensing algorithms including the energy detection [3], cyclostationary detection [4], covariance based detection [5], maximum-minimum eigenvalue detection [6], and signature based detection [7] were proposed to the WRAN. These methods have different advantages and disadvantages. Energy detection is simple and detects signals without prior knowledge of signals. But sensing performance of energy detection method is insufficient in very low SNR and sensitive to noise uncertainty [8]. Cyclostationary detection performs well when SNR is very low, but there is performance degradation due to noise uncertainty. Signature based detection utilizes the sync signal of the ATSC (advanced television systems committee) DTV (Digital Television) signal, so that it can identify ATSC DTV signal, but does not work in very low SNR. Covariance based detection and maximum-minimum eigenvalue detection are not related to noise uncertainty, but these methods cannot detect signal in very low SNR. We propose spectrum sensing algorithm using Bussgang theorem, and the proposed algorithm is not related to noise uncertainty anymore and can meet the requirement of the WRAN.

The proposed algorithm calculates the statistical difference between the Gaussian noise and the primary user signal by applying Bussgang theorem [9]. If the received signal includes the non-gaussian primary user signal, the proportionality of auto-correlation to cross-correlation is different from the proportional constant of the Gaussian signal (the Gaussian noise). Bussgang gaussianity test for stationary series use this approach [10]. Unfortunately, ATSC DTV signal as a primary user signal for WRAN follows the Gaussian distribution in the time domain, so that the proposed method cannot detect ATSC DTV signal in time domain. But ATSC DTV signal does not follow the Gaussian distribution in the frequency domain due to the pilot tone, therefore we can apply proposed sensing method to detect the signal.

This paper is organized as follows. We present the spectrum sensing system model in section II. In section
III, we present a brief description of Bussgang theorem and the spectrum sensing using Bussgang theorem. We describe the proposed sensing algorithm for IEEE 802.22 WRAN, and theoretical analysis of test statistic and threshold in section IV. Simulation results demonstrating the sensing performance of the proposed method in the low SNR is presented in section V. Finally, conclusion is given in section VI.

II. SPECTRUM SENSING SYSTEM MODEL

Assume that there is a single sensor with M receivers (antennas). The aim of the detection problem is to distinguish between the two hypotheses \( H_0, H_1 \). The received signal \( r_i[n] \) through the \( i \)-th receiver is described in the following:

\[
H_0: r_i[n] = w_i[n] \\
H_1: r_i[n] = s[n] * h_i[n] + w_i[n]
\]

where \( s[n] \) is the transmitted signal from a primary user, \( h_i[n] \) is the channel impulse response from the primary user to the \( i \)-th receiver, and \( w_i[n] \) is the white Gaussian noise \( (i=1,2...,M) \). The hypothesis \( H_0 \) assumes that the primary user signal is absent hence only the white Gaussian noise is present in \( r_i[n] \). The hypothesis \( H_1 \) assumes that the primary user signal \( s[n] \) to be detected is present.

The performance measures of a sensing algorithm are the probability of missed detection \( (P_{MD}) \), and the probability of false alarm \( (P_{FA}) \). \( P_{MD} \) is the probability that the sensing algorithm is unable to detect the presence of the primary signal at the hypothesis \( H_1 \). \( P_{FA} \) is the probability that the sensing algorithm decides that the signal \( s[n] \) exists at the hypothesis \( H_0 \). The good sensing algorithm requires low \( P_{MD} \) and \( P_{FA} \) in the low SNR, however there is a trade-off between \( P_{FA} \) and \( P_{MD} \).

III. BUSSGANG THEOREM AND SPECTRUM SENSING USING BUSSGANG THEOREM

Bussgang theorem states that when a real Gaussian stationary process passes through a non-memory non-linear device, the cross-correlation function of input and output is proportional to the auto-correlation function of input \([9]\), that is,

\[
K_g = \frac{E[x[n] \cdot g(x[n+k])]}{E[x[n] \cdot x[n+k]]}
\]

where \( x[n] \) is a real Gaussian stationary process, \( g(\cdot) \) is a non-memory non-linear function and \( K_g \) is the proportional constant. When the input \( x[n] \) is the zero-mean unit-variance Gaussian stationary process, the proportional constant \( K_g \) is described as

\[
K_g = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{x}{x} e^{-x^2/2} dx
\]

where \( g(\cdot) \) is a function of \( g(\cdot) \). If the signal to be detected is not a Gaussian stationary process, there is some difference between the theoretical proportional constant and the sample proportional constant. Based on this difference, we can decide whether or not the objective signal exists. The sample proportional constant \( K_i[\tau] \) at the receiver \( i \) is given by

\[
K_i[\tau] = \frac{\sum_{n=0}^{N-1} \hat{r}_i[n] \cdot g(\hat{r}_i[n+\tau])}{\sum_{n=0}^{N-1} \hat{r}_i[n] \cdot \hat{r}_i[n+\tau]}
\]

where \( \hat{r}_i[n] \) is the normalized version (zero mean, unit-variance) of the received signal at the receiver \( i \), \( \tau \) is a time delay (-\( N \),...,\( N \)), and \( N \) is the number of the received signal sample. The test statistic is defined as

\[
T[\tau] = \sum_{\tau=0}^{M-1} \left( K_i[\tau] - K_g \right)^2
\]

If \( T[\tau] > \lambda \), the signal is present, while if \( T[\tau] \leq \lambda \), the signal is absent where \( \lambda \) is the sensing threshold.

For hypothesis \( H_0 \), there is only the Gaussian noise in the received signal and we can derive the theoretical threshold is decided by the required \( P_{FA} \). If \( \tau \) is zero, the proportional constants are equal to the cross-correlations of the signal and the distorted signal, and follow the Gaussian distribution with the mean \( K_g \) and the standard deviation \( \sigma_g \) based on Central Limit Theorem. The \( T[\tau] \) follows the \( M \)-order chi-squared distribution. The cumulative distribution function of \( T[\tau] \) is in the following

\[
F_T(y) = \int_0^y \frac{1}{\sigma_g^2} \frac{1}{\Gamma(M/2)} u^{M/2-1} e^{-u/2\sigma_g^2} du
\]

where \( \Gamma(\cdot) \) is a gamma function. For the required \( P_{FA} \), the corresponding threshold \( \lambda \) is \( F_T(\lambda) = 1 - P_{FA} \). Finally, we obtain the threshold for the proposed algorithm as

\[
\lambda = F_T^{-1}(1 - P_{FA})
\]

The threshold is determined by required \( P_{FA} \) and used non-memory non-linear function.
IV. SPECTRUM SENSING FOR IEEE 802.22 WRAN

In this section, we describe spectrum sensing method to detect the ATSC DTV signal. There are several primary user signals for the IEEE 802.22 WRAN, but we consider only the DTV signal as a primary user signal. This signal is 8VSB (vestigial sideband modulation) modulated with the sampling frequency $f_s = 21.5244$MHz, and down converted to 5.38MHz, hence the pilot tone of the signal is close to 2.69MHz.

Figure 1 illustrates the block diagram of the proposed spectrum sensing algorithm. We assume there is a single sensor with a single receiver. The received signal is filtered with bandwidth 6MHz, and down converted to the pilot tone at the baseband. The ATSC DTV signal is a Gaussian process because of the pilot tone. Consequently, we filter the DTV signal by narrowband baseband filter with the bandwidth $BW = N_{FFT} / TZ$ to obtain the pilot tone. The filtered pilot tone is decimated by a factor of $floor(f_s / BW)$, and the down sampled signal $x[n]$ is converted to the frequency domain using FFT (fast Fourier transform) of length $N_{FFT}$. $T = 5$ ms sensing time, and $Z$ is a sensing time constant. If $Z = 1, 2, 3, ..$, the total sensing time $TZ$ is 5, 10, 15, ... ms. In the case of $H_0$, the frequency domain signal $X[k]$ is a white Gaussian process. We divide $X[k]$ into real and imaginary part, and normalize the each part for zero-mean unit-variance. The normalized signal $Y_R[k]$ and $Y_I[k]$ are given by

$$Y_R[k] = \frac{X_R[k] - \hat{m}_R}{\hat{\sigma}_R}, \quad Y_I[k] = \frac{X_I[k] - \hat{m}_I}{\hat{\sigma}_I}. \quad (9)$$

where $X_R[k]$ is the real part of $X[k]$, $X_I[k]$ is the imaginary part of $X[k]$, $\hat{m}_R$ and $\hat{m}_I$ are the estimated means of $X_R[k]$, $X_I[k]$, and $\hat{\sigma}_R$ and $\hat{\sigma}_I$ are the estimated standard deviations of $X_R[k]$ and $X_I[k]$. We calculate the each proportional constant $K_R[0]$ and $K_I[0]$ using (5). Finally, the test statistic $T[\tau]$ is defined as

$$T[\tau] = (K_R[\tau] - K_g)^2 + (K_I[\tau] - K_g)^2. \quad (10)$$

The test statistic $T[0]$ is the sum of two zero-mean Gaussian random variable, therefore $T[0]$ follows the 2nd-order chi-squared distribution. The cumulative distribution function of $T[0]$ is given by

$$F_T(x) = 1 - e^{-x/2\sigma_g^2} \quad (11)$$

where $\sigma_g$ is the standard deviation of the $K_R[0]$ and $K_I[0]$, it can be easily derived with the property of sum-process. Finally, we obtain the theoretical threshold for the proposed algorithm as

$$\lambda = -2\sigma_g^2 \cdot \ln(P_{FA}). \quad (12)$$

The threshold is not related to the sensing time, and the noise variance, but the threshold vary with $\sigma_g$.

V. SIMULATION RESULTS

In this section, the sensing performance of the proposed algorithm to detect ATSC DTV signal is considered. The initial signal processing of the signal is based on [11]. We follow the simulation steps and the WRAN spectrum sensing scenario 2 (a single WRAN spectrum sensor with a single receiver) in [12]. Table 1 describes used ATSC A/74 captured DTV files in our simulations. The used parameters are $N_{FFT} = 2048$, $\tau = 0$, $f_s = 21.5244$MHz, $T = 5$ms, $Z = 1, 2, 4$, $BW = N_{FFT} / TZ$, and $P_{FA} = 0.1$.

Figure 2, 3, and 4 show average sensing performances of proposed algorithm over 12 DTV files, and the sensing performance of MME (the maximum-minimum eigenvalue detection) as a function of the SNR at the total sensing time 5, 10, and 20ms. The size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME. At the sensing time 5, 10, and 20ms, the size of the covariance matrix is 10 by 10 in MME.
g(·). Especially, power series functions show good performances among several non-linear functions. Figure 5 demonstrated that when the higher and odd order function is used, the proposed algorithm achieves higher sensing performance. However, the complexity of the proposed algorithm is affected by g(·), therefore we should select the proper g(·) in considering the required sensing performance and the computational complexity.

Figure 6 depicts performance of each 12 DTV files when g(x) = sgn(x)[1 - exp(-x^2/2)]. There are significant differences among 12 DTV signals, because the pilot tones of 12 DTV signals have different frequency offset and power.

### VI. CONCLUSION

In this paper, we proposed the spectrum sensing algorithm using Bussgang theorem for IEEE 802.22 WRAN. In addition we discussed the theoretical analysis of the test statistic and the threshold. Proposed algorithm detects the objective signal using the statistical difference between the Gaussian noise and the primary user signal by applying Bussgang theorem to the received signal. ATSC DTV signal can be detected in frequency domain due to it follows the Gaussian distribution in time domain. Simulation results demonstrate that proposed algorithm can achieve \( P_{MD} = 0.1 \) subject to \( P_{F_1} = 0.1 \) when the sensing time = 20 ms and SNR = -22dB. The non-memory non-linear function determines the sensing performance and the complexity therefore it should be selected after considering required sensing performance and computational complexity. Though the most of the conventional sensing algorithm is affected by the noise uncertainty, the sensing performance of the proposed algorithm is not related to noise uncertainty.

### REFERENCES


