Abstract—In this paper, we investigate the effects of RF impairments and imperfect channel estimation (ICE) on the performance of quadrature amplitude modulation (QAM) for Orthogonal Frequency Division Multiplexing (OFDM) systems employing transmit diversity with two transmit antennas and one receive antenna in a Rayleigh fading environment. We derive the bit error rate (BER) expressions of space-time block codes (STBC)-OFDM system using the Alamouti code. Although the analysis is presented for 16-QAM, it can be easily extended to higher order square QAM constellations. Simulation results are also presented to illustrate the extent of the effects on system performance degradation, with emphasis given to the IEEE 802.16e standard.

I. INTRODUCTION

High performance transceiver designs have been proposed for Orthogonal Frequency Division Multiplexing (OFDM) systems to develop high data rate wireless communication systems due to the compelling advantages over competing technologies. OFDM is a multi-carrier modulation technique in which the available bandwidth is divided among closely spaced but orthogonal frequency-flat sub-channels also referred to as subcarriers. This makes OFDM more robust in frequency-selective fading channels. The technology is proven to be a key technique for achieving needed spectral efficiency and higher data rates requirements for current and future wireless communication systems, and is being used in many wireless applications such as digital video broadcasting (DVB), wireless local area networks (WLAN) and possible next-generation wireless networks such as fixed and mobile WiMax [1], [2].

As the current trend of communication systems demands highly power-efficient and bandwidth-efficient schemes, techniques that provide such desirable properties are considered very valuable in next generation wireless systems. Making use of multiple antennas increases the capacity of the system with the associated higher data rates than single antenna systems [4], [5], [6], [9], [13]. Space-Time coding is a power-efficient and bandwidth-efficient method of communication over fading channels by exploiting the benefits of multiple transmit antennas [3], [6].

The performance of OFDM is known to be severely affected by hardware imperfections and RF impairments such as carrier frequency offset (CFO) and oscillator phase noise [8], [10] with further degradation due to imperfect channel estimation [6], [10].

The effects of phase noise in OFDM have been documented in the literature, including, but not limited to [8], [10]. These works investigated a non-STBC OFDM system with imperfect channel estimation only considered in [10]. The performance of STBC-OFDM is available in the literature. While [9] and [13] investigated STBC-OFDM in a mobile environment, [9] presented only simulation results with no analysis for the BER. The author in [14] assumed perfect knowledge of the channel while [4] and [11] investigated performance gains and trade-offs in STBC-OFDM for higher number of transmit antennas ($N_t \geq 4$). In all these works, the effects of phase noise and imperfect channel estimation is not considered. The work of [2] on the other hand focuses on channel estimation with no phase noise contribution.

This paper extends the work in [10] to a Multi Input Single Output (MISO) communication system by considering STBC-OFDM over two transmit antennas with applications to IEEE802.16e standard. Highlighting the differences with [10], this work assumes a different channel estimation model that is known to be highly effective in fading channels. Furthermore, we investigate the performance of the systems subject to phase noise and imperfect channel estimation using the channel estimation model described within, and derive the analytical BER expressions of the system.

The rest of the paper is organized as follows. The system model is described in section II. Analysis for detection with imperfect channel estimation is presented in Section III. Section IV summarizes the results while the concluding remarks are presented in section V.

II. SYSTEM MODEL

We consider an OFDM system with transmit diversity, in which the total system bandwidth is divided into $N$ equally spaced and orthogonal sub-carriers. The information bits at the transmitter are grouped and mapped into the complex M-QAM symbols. Specifically, we investigate the Alamouti STBC-OFDM system with two transmit antennas and one receive antenna. The Alamouti code is used to encode the transmitted symbols at the two transmit antennas and can be described as follows. During the first time instant, the two symbols $[X_0 \; X_1]$ are transmitted from the two antennas...
simultaneously, with $X_0$ and $X_1$ transmitted from first and second antennas respectively. In the second time instant, the symbols $[-X_1^* \ X_0^*]$ are transmitted simultaneously from the two transmit antennas. This encoding of the transmitted symbol sequence from the transmit antennas is given by the encoding matrix [3], [5]

$$
\theta_2 = \begin{bmatrix}
X_0 & X_1 \\
-X_1^* & X_0^*
\end{bmatrix} \quad (1)
$$

For each transmit antenna, a block of $N$ complex-valued data symbols $\{X(k)\}_{k=0}^{N-1}$ are grouped and converted into a parallel set to form the input to the OFDM modulator, where $k$ is the subcarrier index and $N$ is the number of subcarriers. The modulator consists of an Inverse Fast Fourier transform (IFFT) block. The output of the IFFT at each transmitter is the complex baseband modulated OFDM symbol in discrete time domain and is given by

$$
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}; \quad 0 \leq n \leq N-1 \quad (2)
$$

A. Channel

The channel is modeled by a tapped delay line with channel coefficients that are assumed to be slowly varying such that they are almost constant over the two transmission instants. The channel frequency response for the $k^{th}$ subcarrier is

$$
H(k) = \sum_{p=0}^{L-1} h(p)e^{j2\pi pk/N} \quad (3)
$$

where $h(p)$ is the complex channel gain of the $p^{th}$ multipath component.

B. Phase Noise

The phase noise $\theta(n)$ is modeled as a zero-mean continuous Brownian motion process [8], [10] with variance $\sigma_2^0$. The phase noise increments take the form of a Wiener process, with

$$
\sigma_h k \quad \text{almost constant over the two transmission instants.}
$$

C. Received Signal

The time-domain received signals at the first and second transmission instants at the input to the FFT block are respectively given by

$$
y^0(n) = \left(h_0(n) \circ x_0(n) + h_1(n) \circ x_1(n) + w(n)^0\right)e^{j\theta(n)}
$$

$$
y^1(n) = \left(-h_0(n) \circ x_1^*(n) + h_1(n) \circ x_0^*(n) + w(n)^1\right)e^{j\theta(n)} \quad (4)
$$

where $\circ$ represents linear convolution, subscripts indicate antenna index, and superscripts indicate transmission instant. The complex Gaussian random variable $w(n)$ represents the Additive White Gaussian Noise (AWGNN) term with $\sigma_w^2 = E[|w(n)|^2]$, and $\theta(n)$ is the phase noise.

The frequency-domain received signal can be written as the sum of the desired signal component, the inter-carrier interference (ICI), and the noise term.

$$
\begin{bmatrix}
Y_0 \\
Y_1^*
\end{bmatrix} = \begin{bmatrix}
H_0V_A & H_1V_A \\
H_1^*V_A & -H_0^*V_A
\end{bmatrix} \begin{bmatrix}
X_0 \\
X_1
\end{bmatrix} + \begin{bmatrix}
\sum_{k=0, k \neq l}^{N-1} H_0X_0 V_B \\
\sum_{k=0, k \neq l}^{N-1} H_1X_1 V_B
\end{bmatrix} + \begin{bmatrix}
W_0 \\
W_1^*
\end{bmatrix} \quad (5)
$$

where $V_A$ and $V_B$ are defined as

$$
V_A = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\theta(n)} \quad (6)
$$

$$
V_B(k) = \frac{1}{N} \sum_{n=0}^{N-1} e^{-j2\pi kn/N} e^{j\theta(n)} \quad (7)
$$

The term $V_A$ is common to all subcarriers, and is frequently referred to in the literature as the common phase error (CPE). It can be corrected using pilots or other special techniques [8], [10].

III. DETECTION WITH IMPERFECT CHANNEL ESTIMATION

In the presence of imperfect channel estimation, we assume a channel estimation model such that the channel estimate $\hat{H}$ of the true channel $H$ is given by

$$
\begin{bmatrix}
\hat{H}_0 \\
\hat{H}_1^*
\end{bmatrix} = \begin{bmatrix}
H_0 + \hat{\varepsilon}_0 \\
H_1^* + \hat{\varepsilon}_1
\end{bmatrix} = \begin{bmatrix}
H_0 + \varepsilon_0 \\
H_1 + \varepsilon_1
\end{bmatrix} \begin{bmatrix}
H_0 + \varepsilon_0 \\
H_1 + \varepsilon_1
\end{bmatrix} \quad (8)
$$

where $\varepsilon_0$ and $\varepsilon_1$ are the errors in the channel estimate from the first and second transmit antennas respectively, and are modeled as independent zero-mean complex Gaussian random variables with variances $2\sigma_2^0$ and $2\sigma_2^1$ respectively.

We assume that the CPE term $V_A$ is effectively estimated and fully compensated using pilots that are continuously inserted within the OFDM symbols. The detected symbols $\hat{X}$ after equalization are given by

$$
\hat{X} = \hat{H}^H Y
$$

$$
\begin{bmatrix}
\hat{X}_0 \\
\hat{X}_1^*
\end{bmatrix} = \begin{bmatrix}
H_0^2 + \varepsilon_0^2 & H_1^2 + \varepsilon_1^2 & H_0^* \varepsilon_0 + H_1^* \varepsilon_1 \\
H_1^2 + \varepsilon_1^2 & -H_0^2 - \varepsilon_0^2 & H_0^* \varepsilon_1 - H_1^* \varepsilon_0
\end{bmatrix} \times
\begin{bmatrix}
\sum_{k=0, k \neq l}^{N-1} H_0X_0 V_B \\
\sum_{k=0, k \neq l}^{N-1} H_1X_1 V_B
\end{bmatrix} + \begin{bmatrix}
H_0^2 + \varepsilon_0^2 \\
H_1^2 + \varepsilon_1^2
\end{bmatrix} \begin{bmatrix}
H_0^* \varepsilon_0 + H_1^* \varepsilon_1 \\
H_0^* \varepsilon_1 - H_1^* \varepsilon_0
\end{bmatrix} \times
\begin{bmatrix}
W_0 \\
W_1^*
\end{bmatrix} \quad (9)
$$
The variance of $\Psi$ is given by
\[
\sigma^2_\Psi = E[|\Psi|^2]
\]
\[
= \sigma^2_{H_0}E_s \left[ \sum_{i=0}^1 \sigma^2_{\varepsilon_i} \right]
+ \sigma^2_{H_1}E_s \left[ \sum_{i=0}^1 \sigma^2_{\varepsilon_i} \right]
\] (14)

Similarly, the variance of the ICI term $\tilde{\beta}$ is given by
\[
\sigma^2_{\tilde{\beta}} = E[|\tilde{\beta}|^2]
\]
\[
= E_sE_{\Phi} \left[ \sum_{i=0}^1 \sigma^2_{H_i} \right]
+ \sum_{i=0}^1 \sigma^2_{\varepsilon_i}
\] (15)

where $E_{\Phi}$ is given by [8], [10]
\[
E_{\Phi} = [1 - \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \Phi_{\Delta n m}(1)]
\] (16)

and $\Phi_{\Delta n m}(1)$ is the characteristic function of the phase noise increment evaluated at 1.

We present the bit error rate analysis for the case of 16QAM modulation using Gray code mapping for $(b_1b_2b_3b_4)$. It is important to note that although the presentation is only for 16QAM, the following analysis is valid for all square QAM constellations. The conditional BER for bit $b_1$, condition on $\hat{H}_0, \hat{H}_1$ is given by
\[
P_e(b_1|\hat{H}_0, \hat{H}_1) = \frac{1}{2} \left[ Pr(\tilde{X}_I < 0|X_I = d, \hat{H}_0, \hat{H}_1)
+ Pr(\tilde{X}_I < 0|X_I = 3d, \hat{H}_0, \hat{H}_1) \right]
\] (17)

where $X_I, \tilde{X}_I$ are the real parts of $X, \tilde{X}$ respectively. Similarly, the conditional bit error probability for bit $b_3$ is
\[
P_e(b_3|\hat{H}_0, \hat{H}_1) = \frac{1}{2} \left[ Pr(|\tilde{X}_I| > 2d|X_I = d, \hat{H}_0, \hat{H}_1)
+ Pr(|\tilde{X}_I| < 2d|X_I = 3d, \hat{H}_0, \hat{H}_1) \right]
\] (18)

Rewriting (17) and (18) as a sum of Q-functions, and substituting for $d$ as a function $E_0$, the conditional BER for bit $b_1$ can be written as
\[
P_e(b_1|\hat{H}_0, \hat{H}_1) = \frac{1}{2} \left[ Q\left( \sqrt{\frac{d^2}{\sigma^2_{\Psi} + \sigma^2_{\beta} + \sigma^2_W}} \right)
+ Q\left( \sqrt{\frac{9d^2}{\sigma^2_{\Psi} + \sigma^2_{\beta} + \sigma^2_W}} \right) \right]
\]
\[
= \frac{1}{2} \left[ Q\left( \sqrt{\frac{(2E_0/5)[|H_0|^2 + |H_1|^2]}{\sigma^2_{\Psi} + \sigma^2_{\beta} + \sigma^2_W}} \right)
+ Q\left( \sqrt{\frac{9(2E_0/5)[|H_0|^2 + |H_1|^2]}{\sigma^2_{\Psi} + \sigma^2_{\beta} + \sigma^2_W}} \right) \right]
\] (19)
and for bit $b_3$ is given by

$$P_e(b_3|\hat{H}_0, \hat{H}_1) = \frac{1}{2} \left[ Q \left( \sqrt{\frac{9d^2}{\sigma^2_{\phi} + \sigma^2_{\beta} + \sigma^2_W}} \right) + Q \left( \sqrt{\frac{d^2}{\sigma^2_{\phi} + \sigma^2_{\beta} + \sigma^2_W}} \right) ight. $$

$$+ Q \left( \sqrt{\frac{2d^2}{\sigma^2_{\phi} + \sigma^2_{\beta} + \sigma^2_W}} \right) - Q \left( \sqrt{\frac{25d^2}{\sigma^2_{\phi} + \sigma^2_{\beta} + \sigma^2_W}} \right) \right]$$

From which the SNR $\gamma$ is given by

$$\gamma = \frac{(|H_0|^2 + |H_1|^2)2E_b/5}{(\sigma^2_{\phi} + \sigma^2_{\beta} + \sigma^2_W)}\right]$$

and follows a Chi-square distribution [12], with probability density function (PDF) given by

$$p(\gamma) = \frac{1}{2\gamma^{\frac{1}{2}}} e^{-\frac{\gamma}{2\gamma^{\frac{1}{2}}} \right]$$

Due to the symmetry of square M-QAM constellations, the BER for the in-phase and quadrature bits are equal such that $P_e(b_1) = P_e(b_2)$ and $P_e(b_3) = P_e(b_4)$. Therefore the average BER is obtained by averaging the conditional BER of $b_1$ and $b_3$ over the PDF of the SNR $\gamma$. The average BER is therefore given by

$$P_e = \frac{1}{2} \int_0^\infty \left[ P_e(b_1|\hat{H}_0, \hat{H}_1) + P_e(b_3|\hat{H}_0, \hat{H}_1) \right] p(\gamma) d(\gamma)$$

\textbf{IV. RESULTS}

\textbf{A. Simulation Parameters}

The simulation parameters are presented in Table-I, making use of the proposed channel parameters and bandwidth specifications in the IEEE 802.16e standard, as presented in [1], [2].

\textbf{B. Discussion of Results}

Fig.1 shows the effect of phase noise on the performance of Alamouti STBC-OFDM. It is observed that even at high SNR, the error floor due to ICI is quite evident. In Fig.2, the curves show the effect of imperfect channel estimation with $\sigma^2 = [0.0, 0.02, 0.04, 0.06]$ on system performance. The results demonstrate that good channel estimates are very important in determining appreciable system performance. Fig.3 illustrates the impact of low quality oscillator on system performance. The combined effects of phase noise and channel estimation errors on the system is shown in Fig.4, while Fig.5 illustrates the effect of poor channel estimates on system performance.

\textbf{V. CONCLUDING REMARKS}

The effects of phase noise and imperfect channel estimation on the performance of Alamouti coded OFDM in fading channels are analyzed. The transmit diversity scheme is shown to be severely affected by the phase noise and channel estimation errors as well. The BER curves show that the system impairments cause significant performance loss as a consequence of these negative effects in the form of ICI.

\textbf{REFERENCES}

Fig. 2. BER of 16QAM STBC-OFDM Transmit Diversity with Imperfect Channel Estimation (ICE)

Fig. 3. BER vs. Quality of Local Oscillator for at fixed SNR: 16QAM STBC-OFDM Transmit Diversity

Fig. 4. BER of 16QAM STBC-OFDM Transmit Diversity with Phase Noise & Imperfect Channel Estimation (ICE)

Fig. 5. BER vs. Quality of Channel Estimate for at fixed SNR: 16QAM STBC-OFDM Transmit Diversity


