A NOVEL METHOD OF DOPPLER SHIFT ESTIMATION FOR OFDM SYSTEMS

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ABSTRACT

Doppler shift estimation has been deployed to improve the performance of Orthogonal Frequency Division Multiplexing (OFDM) systems. A novel method of Doppler shift estimation for OFDM systems in time varying and frequency selective fading channels has been proposed and investigated. The proposed method is a frequency domain scheme that utilizes pilot subcarriers, which are adopted in most practical OFDM systems. An equation consisting of the Doppler shift has been derived from the ratio of two autocorrelation sample points at the receiver so that the Doppler shift estimation can be retrieved. Since the computation for solving the equation is formidable, the proposed algorithm has also been further simplified by using the optimal polynomial approximation to the zero order Bessel function of the first kind. Computer simulation demonstrates that the proposed algorithm can estimate the Doppler shift accurately within a wide range of Doppler shift.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an efficient high data rate transmission technique for wireless communication [1]. OFDM presents advantages of high spectrum efficiency, simple and efficient implementation by using the fast Fourier transform (FFT) and the inverse Fast Fourier Transform (IFFT), mitigation of inter-symbol interference (ISI) by inserting cyclic prefix (CP) and robustness to frequency selective fading channel. Doppler shift estimation techniques have been employed to further improve the OFDM system performance. For instance, the Doppler shift is used in the process of optimizing the interleave length for the adaptive modulation and coding bit-interleaved coded OFDM (AMC-BIC-OFDM) system to reduce the receiving delay [2]. It is also used in adaptive channel estimation with a wide range of user velocities and used to obtain the coefficients of the Wiener filtering for the adaptive two-dimensional channel estimator [3].

There are four major kinds of Doppler shift estimation which are based on channel estimates [4], level crossing rate (LCR) [5], maximum likelihood function [6] and correlation function [7], respectively. In [4], A Doppler shift estimation method based on channel estimates has been proposed, which is effective at low Doppler shift. However, the accuracy of the Doppler shift estimation will reduce if the channel estimation is not accurate enough. In [5], an improved level crossing rate estimation method has been proposed. This method is composed of a Doppler adaptive noise suppression procession and a conventional LCR estimator. The improvement of the accuracy of the estimation is due to the increase of the estimator complexity. In [6], an estimator based maximum likelihood function has been proposed and it has high accuracy of estimation. However, it has high computation complexity and can not be realized in practical systems. In [7], the autocorrelation of the cyclic prefix (CP) of OFDM system is used to estimate the Doppler shift, the estimated Doppler shift is often overestimated when the Doppler shift is small, especially at low SNR.

This paper proposed a novel method of Doppler shift estimation and it is based on the ratio of two autocorrelation sample points of the received signal in frequency domain. Since time-varying fading channel can cause inter-carrier interference (ICI) in OFDM systems, we take ICI into consideration and derive the autocorrelation function of the received signal at pilot subcarrier. Since the form of the autocorrelation function is too complicated, we design an appropriate pilot pattern, i.e., the comb-type [11], to simplify the form significantly. Then, an equation composed of the Doppler shift can be obtained based on the ratio of two autocorrelation sample points of the received signal. Since solving the equation requires large computation, we also propose a simplified algorithm by using the optimal polynomial approximation to the zero order Bessel function of the first kind and it can reduce the computation complexity significantly.

This paper is organized as follows. Section II introduces the OFDM system model. The proposed
algorithm of Doppler shift estimation is described in Section III. Computer simulation results of the proposed algorithm are discussed in Section IV followed by conclusion in Section V.

II. SYSTEM MODEL

We adopted the OFDM system model in [8] without loss of generality. For \( N \) carriers in the OFDM system, the transmitted signal \( x(i,n) \) in time domain after inverse Fast Fourier Transform (IFFT) is given by

\[
x(i,n) = \frac{1}{N} \sum_{k=0}^{N-1} X(i,k) \exp\{j2\pi nk / N\}
\]

where \( X(i,k) \) denotes the transmitted signal in frequency domain at the \( k \)-th subcarrier in the \( i \)-th OFDM symbol.

A few pilots are inserted into each OFDM symbol and the set of the indexes of pilot subcarriers is \( \beta \). The values of pilots in the set \( \beta \) are assumed to be the same. The value of \( X(i,k) \) (including the pilots) is set according to the modulation constellation. We assume that the data \( X(i,k) \) is normalized, i.e., \( |X(i,k)|^2 = 1 \). For instance, \( X(i,k) = \pm 1 \) and \( X(i,k) = (\pm 1 \pm \sqrt{-1})/\sqrt{2} \) for BPSK and QPSK, respectively. It is assumed that \( E[X(i,k)] = 0 \), \( k \not\in \beta \) and \( E[X(i,k)X^* (i_z,k)] = 0 \), \( i \neq i_z, k \not\in \beta \), where \( E(\cdot) \) denotes expectation, \( (\cdot)^* \) denotes conjugate. A cyclic prefix (CP) is inserted into each OFDM symbol prior to the transmission and the CP is removed before the Fast Fourier Transform (FFT) process at the receiver. The received signal \( Y(i,k) \) in frequency domain after FFT can be written as [9]

\[
Y(i,k) = X(i,k)g(i,k,k) + \sum_{j=0, j \neq k}^{N-1} g(i,k,j)X(i,j) + W(i,k)
\]

where \( W(i,k) \) denotes the AGWN with zero mean and variance \( \sigma_w^2 \),

\[
g(i,p,q) = \frac{1}{N} \sum_{n=0}^{N-1} h(i,n,\tau) e^{-j2\pi n(p-q) / N} e^{-j2\pi q\tau_i}
\]

where \( h(i,n,\tau) \) is the channel impulse response in time domain of the \( l \)-th path at the \( n \)-th sample point within the \( i \)-th symbol time, \( \tau_l \) is the channel delay of the \( l \)-th path, \( L \) is the number of resolvable paths. It is assumed that different paths \( h(i,n,\tau_l) \) are independent and the power of the \( l \)-th path is \( \sigma_l^2 \). The channel is normalized so that \( \sigma_l^2 = \sum_{l=1}^{L} \sigma_l^2 = 1 \). The channel autocorrelation function is given by

\[
E[h(i_n,\tau_l)h^*(i_z,n_z,\tau_l)] = \sigma_l^2 \delta(\tau_l - \tau_l) J_0(2\pi f_d T_{sym}[(i_i - i_z) + (n_i - n_z)/N_{sym}])
\]

where \( \delta(\cdot) \) is the Kronecker delta function, \( J_0(\cdot) \) is the zero order Bessel function of the first kind, \( f_d \) is the maximum Doppler shift, \( T_{sym} \) is the during time of an OFDM symbol including the CP, \( N_{sym} \) is the length of an OFDM symbol including the CP in unit of sample point, \( (\cdot)^* \) denotes conjugate. In addition, the item \( \sum_{j=0, j \neq k}^{N-1} g(i,k,j)X(i,j) \) in (3) is the inter-carrier interference (ICI).

III. DOPPLER SHIFT ESTIMATION

In this section, we propose a Doppler shift estimation method based on the autocorrelation function of the received signal. By designing the set of pilot subcarriers \( \beta \), the form of the autocorrelation function of the received signal can be simplified significantly. Then, we obtain an equation consisting of the Doppler shift and the Doppler shift can be retrieved by solving the equation. Since the computation of solving the equation is formidable, we propose a simplified method to calculate the Doppler shift by approximating the Bessel function optimally.

A. The proposed original Doppler shift estimation

The autocorrelation function of the received signal \( Y(i,k'), k' \in \beta, R_{yy}(i,k') \), is depicted in Appendix A and it is given by


\[ R_{yy}(l,k') = E[Y(l,k')Y^*(l+1,k')] \]

\[ \frac{1}{N^2} \sum_{n_1=0}^{N_c-1} \sum_{n_2=0}^{N_c-1} J_{n_1}J_{n_2} e^{-j \frac{2\pi}{N_c} (n_1-x(n_2-x))} + \sigma^2 e^{-j \frac{2\pi}{N_c} (n_1-x(n_2-x))} + \sigma^2 \]

\[ = \frac{1}{N^2} \sum_{n_1=0}^{N_c-1} \sum_{n_2=0}^{N_c-1} J_{n_1} \left[ 2\pi f_d T_{sym} \left( l + \frac{n_1-n_2}{N_{sym}} N_P \right) \right] \]

Due to limited space, we omit the details of derivation of formula (6). When \( l \) is equal to 1 or 3, we have the following ratio

\[ C = R_{yy}(1) / R_{yy}(3) \]

\[ = \frac{\sum_{n_1=0}^{G_c-1} \sum_{n_2=0}^{G_c-1} J_0 \left[ 2\pi f_d T_{sym} \left( 1 + \frac{n_1-n_2}{N_{sym}} N_P \right) \right]}{\sum_{n_1=0}^{G_c-1} \sum_{n_2=0}^{G_c-1} J_0 \left[ 2\pi f_d T_{sym} \left( 3 + \frac{n_1-n_2}{N_{sym}} N_P \right) \right]} \]

Thus, the Doppler frequency shift \( f_d \) can be obtained by solving the above equation (7). In addition, we refer to the Doppler shift estimator expressed by (7) as \( f_{d, \text{org}}(C) \).

**B. The proposed simplified algorithm**

Since solving equation (7) containing the term \( J_0(x) \) requires substantial computation and it is formidable for a mobile handset in practice, a simplified algorithm by the approximation of \( J_0(x) \) is therefore proposed. We want to find the best polynomial approximation of the function \( J_0(x) \) over an interval \( x \in [a, b] \), in the sense that

\[ \int_a^b |f(x) - J_0(x)|^2 dx \]

is minimized for a polynomial \( f(x) \) of degree \( p-1 \), i.e.,

\[ f(x) = a_0 + a_1x + \cdots + a_{p-1}x^{p-1} \]  

In this paper, we choose the interval to be \([0, 2]\) and \( p = 3 \). Using the orthogonality theorem \([12]\), we can obtain the normal equation

\[ Ra = b \]

where \( R \) is a 3 by 3 matrix with each entry \( r_{ij} = \int_0^2 x^i J_0(x) dx = \frac{2i+1}{i+1} \), \( a = [a_0 a_1 a_2]^T \), \( b = [b_0 b_2] \) with each entry \( b_i = \int_0^2 x J_0(x) dx \), \( i = 0, 1, 2 \), \( (\cdot)^T \) denotes transpose. Solving equation (9), we can obtain the coefficients \( a = [1.0183 -0.9904 -0.1546]^T \). Therefore, the best approximation polynomial of degree 2 is

\[ f(x) = 1.0183 - 0.9904x - 0.1546x^2 \]  

(10)

![Figure 1. The optimal polynomial approximation to \( J_0(x) \), \( f(x) = 1.0183 - 0.9904x - 0.1546x^2 \), versus \( J_0(x) \).](image)

Figure 1 shows the best approximation polynomial \( f(x) \) versus \( J_0(x) \). It is observed that \( f(x) \) approaches \( J_0(x) \) well within the range \( x \in [0, 2] \). Replacing \( J_0(\cdot) \) in equation (7) by \( f(\cdot) \), we have...
By simple manipulation of equation (11), the Doppler frequency shift can be expressed in a close-form expression given by

\[
f_{d,\text{simp}} = \frac{0.0994(1-3C) + \sqrt{0.00988(1-3C)^2 + 0.3149(C-1)F(C,G_{ap},N_p,N_{sym})}}{0.3092 \pi T_{sym} \cdot F(C,G_{ap},N_p,N_{sym})}
\]

(12)

where

\[
F(C,G_{ap},N_p,N_{sym}) = (C-1)(G_{ap}^2-1)N_p^2/(3N_{sym}^2) + 2(9C-1)
\]

The Doppler shift can be evaluated directly by the close-form expression (12) if the ratio \( C \) is known, which reduces the computation complexity significantly in comparison with the equation (7).

For the sake of performance comparison, we define the mean value of the estimated Doppler shift, \( \tilde{f}_d \), and the normalized mean square error (NMSE) of the estimated Doppler shift as

\[
\tilde{f}_d = \frac{1}{N_s} \sum_{i=0}^{N_s-1} \tilde{f}_d(i)
\]

\[
\gamma_{\text{NMSE}} = \frac{1}{N_s} \sum_{i=0}^{N_s-1} |\tilde{f}_d(i) - f_d(i)|^2 / f_d^2
\]

(15)

(16)

where \( N_s \) is the number of estimation groups and one estimation group produces one estimated Doppler shift value, \( \tilde{f}_d \) is the estimated Doppler shift given by (7) or (12).

**IV. SIMULATION RESULTS**

Computer simulation based on the system model described in Section II has been deployed to investigate the performance of the proposed algorithm for the Doppler shift estimation. We employ the channel model of COST207 [10] having 6 numbers of paths and the maximum delay spread of 10 \( \mu s \). The number of the subcarriers of the OFDM system, \( N \), is equal to 2048 and the CP length is equal to 256 sample points. So the length of an OFDM symbol including the CP in unit of sample point, \( N_{sym} \), is equal to 2304. The bandwidth of the system is 20 MHz so that \( T_{sym} = 115.2 \) \( \mu s \) and the CP length \( T_{CP} = 12.8 \) \( \mu s \). The pilot pattern is comb-type and the number of pilots in one OFDM symbol is 8 so that the gap between two adjacent pilots, \( G_{ap} \), is 256. The transmitted signal is QPSK modulated.
Figure 3. The mean value of the estimated Doppler shift for the two estimators, $f_{\text{d,org}}$ and $f_{\text{d,simp}}$, versus the true Doppler shift $f_d$.

Figure 4. The normalized mean square error (NMSE) of the estimated Doppler shift for the estimator $f_{\text{d,simp}}$ versus the average length of estimation group $B$, for $f_d = 300$ Hz and 600 Hz, respectively.

Figure 3 depicts the computer simulation result for the mean value of the estimated Doppler shift for two estimators, $f_{\text{d,org}}$ and $f_{\text{d,simp}}$, at the SNR values of 0, 5 and 15 dB, respectively. The average length of one estimation group $B$ is equal to 120, which corresponds to 0.0138s. The number of estimation groups $N_s$ is chosen to be 1000. It can be observed that the mean value of the estimated Doppler shift is not sensitive to SNR for the two estimators, i.e., $f_{\text{d,org}}$ and $f_{\text{d,simp}}$. Both the estimator $f_{\text{d,simp}}$ and $f_{\text{d,org}}$ provide reliable estimation of the Doppler shift ranging from 100 Hz to 800 Hz.

Figure 4 shows the normalized mean square error (NMSE) of the estimated Doppler shift for the estimator $f_{\text{d,simp}}$ versus the average length of estimation group $B$, for $f_d = 300$ Hz and 600 Hz, respectively. The number of estimation groups $N_s$ is chosen to be 1000. It is observed that the NMSE of the estimated Doppler shift decreases when $B$ increases from 50 to 250 for $f_d = 300$ Hz and 600 Hz. The NMSE of the estimated Doppler shift decreases insignificantly when increasing SNR. For instance, the NMSE for $f_d = 300$ Hz decreases from 0.036 to 0.028 when SNR increases from 0 dB to 15 dB for $B = 100$ OFDM symbols. In addition, the NMSE decreases significantly when $f_d$ increases from 300 Hz to 600 Hz. One reason is that the gradient of $f_{\text{d,simp}}(C)$ at 300 Hz is far bigger than the gradient at 600 Hz, which can be found in Figure 2.

V. CONCLUSION

In this paper, a novel algorithm of Doppler shift estimation for OFDM systems over frequency dispersive fading channels has been proposed and investigated. We compare the proposed original algorithm with the simplified algorithm by numerical method and computer simulation. Both numerical result and computer simulation show that the performance of the proposed simplified algorithm is similar to that of the proposed original algorithm. Computer simulation demonstrates the proposed simplified algorithm can estimate the Doppler shift accurately with low estimation latency within a large range of Doppler shift. Therefore, the proposed simplified algorithm can readily be adopted in practical OFDM systems.

APPENDIX A

The autocorrelation function of the received signal $Y(i,k'), k' \in \beta$, $R_{\gamma\gamma}(l, k')$, is given by

$$R_{\gamma\gamma}(l, k') = E[Y(i,k')Y^*(i+l,k')]$$

$$= E \left[ X(i,k')g(i,k',k') + \sum_{k=k'-N+-1}^{k'-1} g(i,k',k)X(i,k)+W(i,k') \right]$$

$$\left[ X(i+l,k')g(i+l,k',k') + \sum_{k=k'+i+l}^{k'+i+l} g(i+l,k',k)X(i+l,k)+W(i+l,k') \right]$$

(A-1)

We study (A-1) for two cases, i.e., $l = 0$ and $l > 0$. When $l = 0$, the formula (A-1) can be further derived as...
Secondly, we compute $E \left[ \left| \sum_{k,\beta} g(i, k', k) X(i, k) \right|^2 \right]$ similar to the derivation of (A-3). We have 
\[
R_{xy} \left( 0, k' \right) = E \left\{ \sum_{k,\beta} g(i, k', k) X(i, k) + \sum_{k,\beta} g(i, k, k') X(i, k) \right\} + W(i, k')^2
\]
\[
= E \left[ \left| \sum_{k,\beta} g(i, k', k) X(i, k) \right|^2 \right] + E \left[ \left| \sum_{k,\beta} g(i, k, k') X(i, k) \right|^2 \right] + \sigma_y^2
\]
\[
(A-2)
\]
Firstly, we compute $E \left[ \left| \sum_{k,\beta} g(i, k', k) X(i, k) \right|^2 \right]$ in (A-2). 
\[
E \left[ \left| \sum_{k,\beta} g(i, k', k) X(i, k) \right|^2 \right] = \sum_{k,\beta} \sum_{n=0}^{N-1} \sum_{\alpha=0}^{N-1} E \left\{ h(i, n, \tau_c) h^\dagger(i, n, \tau_c) \right\} e^{-j2\pi \alpha (k-k')/N} X(i, k)
\]
\[
= \sum_{k,\beta} \sum_{n=0}^{N-1} \sum_{\alpha=0}^{N-1} E \left\{ h(i, n, \tau_c) h^\dagger(i, n, \tau_c) \right\} e^{-j2\pi \alpha (k-k')/N} X(i, k)
\]
\[
(A-3)
\]
It is noted that we have the assumption that the values of pilots in the set $\beta$ are same, which is described in section II. Since $X(i, k) = 1$, then $X(i, k_1) X(i, k_2) = 1$, $k_1, k_2 \in \beta$.

Therefore (A-3) can be further derived as 
\[
E \left[ \left| \sum_{k,\beta} g(i, k', k) X(i, k) \right|^2 \right] = \frac{1}{N^2} \sum_{k,\beta} \sum_{n=0}^{N-1} \sum_{\alpha=0}^{N-1} \sum_{\beta=0}^{N-1} E \left\{ h(i, n, \tau_c) h^\dagger(i, n, \tau_c) \right\} e^{-j2\pi \alpha (k-k')/N} X(i, k)
\]
\[
= \frac{1}{N^2} \sum_{k,\beta} \sum_{n=0}^{N-1} \sum_{\alpha=0}^{N-1} \sum_{\beta=0}^{N-1} \sigma_i^2 j_o \left( 2 \pi f_d T_{sym} \left( \frac{N}{N} - i - j + \frac{n_i - n_j}{N_{sym}} \right) \right) e^{-j2\pi \alpha (k-k')/N} X(i, k)
\]
\[
(A-4)
\]
Secondly, we compute $E \left[ \left| \sum_{k,\beta} g(i, k', k) X(i, k) \right|^2 \right]$ in (A-2). 

Similar to the derivation of (A-3), we have 
\[
E \left[ \left| \sum_{k,\beta} g(i, k', k) X(i, k) \right|^2 \right] = \frac{1}{N^2} \sum_{n_i=0}^{N-1} \sum_{n_j=0}^{N-1} \sum_{k,\beta} J_o \left( 2 \pi f_d T_{sym} \frac{n_i - n_j}{N_{sym}} \right) e^{-j2\pi \alpha (n_i - n_j - k')/N}
\]
\[
(A-5)
\]
Thus, we have obtained $R_{xy}(0, k')$ by substituting (A-4) and (A-5) into (A-2). When $l > 0$, the formula (A-1) can be derived as 
\[
R_{xy}(l, k') = \frac{1}{N^2} \sum_{n_i=0}^{N-1} \sum_{n_j=0}^{N-1} \sum_{k,\beta} J_o \left( 2 \pi f_d T_{sym} \left( l + \frac{n_i - n_j}{N_{sym}} \right) \right)
\]
\[
(A-6)
\]
REFERENCES


The following Reviews are available for this paper:

- The authors propose a Doppler shift estimation method for OFDM systems. The topic has been discussed in many papers for many years. The proposed method is clear and reasonable. However, the authors should compare the proposed method with the existed methods to show its advantages.

- Doppler shift estimation has been deployed to improve the performance of Orthogonal Frequency Division Multiplexing (OFDM) systems. A method of Doppler shift estimation for OFDM systems in time varying and frequency selective fading channels has been proposed and investigated. Since there have been many methods proposed on this topic in the literature, it is recommended that the authors need to compare with other methods.

- The paper investigates the dopple shift estimation issue for OFDM systems. The proposed method is based on the channel autocorrelation function model and the second order statistics. The method takes advantage of the pilot in OFDM systems and the approximation of Bessel function to reduce the complexity. 1. The method heavily depends on the channel autocorrelation function model (4), in practical channels, (4) may not be valid. 2. From Fig.4, the estimation performance depends on the gradient of true dopple shift. When Dopple shift is not so big (Less than 300Hz), the performance is not good enough even the SNR is high (15dB).